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A MODEL FOR PRISON DISTURBANCES

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PRISON disturbances are serious events: they involve danger to staff and prisoners, sometimes even loss of life, and set back the progress of institutional development. Fortunately, however, disturbances are rare and it is this which makes systematic analysis of them difficult—indeed, whilst they may provoke public concern and an inquiry, as after the Dartmoor riot in 1932 (H.M.S.O., 1932), disturbances in one institution are seldom seen as being related to similar events elsewhere, with the result that they are often explained in terms of particular local conditions which happened to prevail at the time. However, Schrag (1960), in a review of riots, has shown that common features are discernible but he highlights the problems of foreseeing them: " Each riot began as a sudden flare-up of violence. In most cases there was no perceptible forewarning, though a state of heightened tension and anxiety was widely recognised." The mixture of tension and suddenness has been most graphically expressed by Fox (1971): " The way to make a bomb is to build a strong perimeter and generate pressure inside "; but he too observes the problem that " finding valid, consistent and reliable information as to why prisoners riot defies most standardised methods of gathering data on human behaviour". The problem, simply, is that conventional statistical techniques are generally unsuited to analysing sudden, discontinuous events: however, a branch of mathematics, known as catastrophe theory (Thom, 1972) gives an account of just this kind of event. Although the early applications of catastrophe theory were primarily in the physical and biological sciences, the theory has lately been applied in the social sciences (Isnard and Zeeman, 1974) and the purpose of this paper is to see what light catastrophe theory is able to cast on prison disturbances.

Basic Properties a Model Should Have

Clearly any model of prison riots should aim to show the nature of the relationship between *tension* and *disorder*, within the general proposition that *more tension is associated with more disorder*. Another factor commonly held to be critical in institutional stability (although in general the evidence is of uneven quality) is the level of communication in the prison, the personal relationships between staff and prisoners and amongst themselves and any

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polarisation which takes place: we may group this under the overall heading of *alienation* and say that *increased alienation is often but not always associated with increased disorder*; indeed, if prisoners were kept separate in their cells all day—that is, highly alienated—the opportunities for generalised disorder would be, of course, much reduced.

In addition to the components of tension and alienation, we will also need to give some account of, for example, the effects of events like a workshop or food dispute which are usually resolved without undue fuss but *can* have an effect out of all proportion to their seriousness, when they are referred to as " triggering " a disturbance. Equally, external events, like the activities of a prisoners' rights group, can have a similar triggering effect, sometimes on several institutions simultaneously. These internal and external events may be seen as *noise* and the impact this noise will have will depend on the reactivity or volatility of the institution. It can be shown (Marriage, 1977) that certain objective parameters of the prisoner population, probably best expressed as the proportion of younger men who concentrate on relatively serious offences, have a marked impact on the likelihood of a serious prison disturbance: these population characteristics may, therefore, be regarded as contributing towards the volatility of the institution and the total amount of noise, or the *noise level* will be a function of both the noise and volatility of the population.

One further aspect of institutions needs to be taken into account in our model. However well-ordered the regime, many, if not most, people will find it monotonous and actively set about seeking and generating stimulation. Whilst this is a general effect, it is especially so for some prisoners (Hare, 1970) and we may safely assume that if prison life becomes " too quiet " there will be a marked tendency on behalf of probably both staff and prisoners to liven things up a bit, to the point where staff particularly, but also prisoners, will naturally react to quieten down the regime a little: in this way the institution can be seen to have a homeostatic tendency which avoids the extremes of " quiet " and " disturbance " in favour of something in between.

Description of the Model

Since we have three main variables—disorder, tension and alienation—the model will be three-dimensional and show discontinuities corresponding to outbreaks of disorder; because of the nature of our postulates about tension and alienation, tension can be identified as the " normal " factor and alienation as the " splitting " factor in a cusp-catastrophe which can be represented as a surface, G. This is illustrated in Figure 1 which shows both the three-dimensional surface and the same surface mapped on two dimensions. It is not necessary to know the underlying mathematics of the model in order to appreciate its significance: because the surface overlaps itself, a catastrophic discontinuity can be the result of a continuous, even gentle change in tension or alienation. Consider a path P (dotted lines in Figure 1), where an institution at a certain constant level of alienation is experiencing continuously rising tension; at first all is quiet and only a slight rise in disorder occurs but after the point S, because the surface has two levels, the possibility arises of "jumping" to the upper surface. However, in a situation of no noise, the institution proceeds, with increasing tension, to the point R beyond which only the upper surface is possible and a sudden increase in disorder takes place. This sudden jump gives catastrophe theory its name. It follows also, of course, that if we reverse the path, disorder will subside only slightly until the point S where there will be a sudden drop as calm returns. Thus, *because the surface overlaps itself*, the model describes the possibilities of discontinuous jumps and "artificial" or "unnatural" levels of disturbance and quietness—this last corresponding to the physical phenomenon of super-cooling.

Mathematical Description of the Model

To model the varying states of the institution accurately, we might use, first, a high-dimensional space, X, to represent all possible states; secondly, a *dynamic* on X representing the human relationships and corresponding to the homeostatic tendency described above; and, thirdly, *noise*, strictly stochastic noise, continually disturbing the equilibrium. Generally the dynamic will be able to restore the equilibrium but exceptionally the noise will be sufficient to carry the system past a threshold—represented in Figure 1 as the interleaving sheet T—so that the dynamic, instead of restoring the equilibrium, takes the system to a new equilibrium level—the change representing, for instance, a sudden outbreak of disorder.

From the classification theory of catastrophe theory (Thom, 1972) it can be shown that G and T together form a smooth surface equivalent [1] to the cusp-catastrophe surface and given by the equation:

$$d^3 = t + ad$$

where d, t and a measure disorder, tension and alienation respectively, in relation to an origin at the cusp point. The graph G is the part given by the $3d^2 \geqslant a$, the threshold surface T is the part given by $3d^2 < a$, and the fold curve between them is given by $3d^2 = a$. The fold-curve is therefore the boundary of G and the projection of the fold curve on to the tension-alienation plane, C, is the cusp, given by the equation:

$$27t^2 = 4a^3$$

The graph G is single-sheeted outside the cusp: so that, at low levels of alienation, disorder is a continuously increasing function of tension. Similarly, when alienation is high but tension is low, there is a single range of low disorder, labelled "quiet" in Figure 1; if, however, both alienation and tension are high, there is a single level of high disorder which we have labelled "disturbed". But the graph is double-sheeted inside the cusp.

[1] Equivalent means precisely that there is a smooth map of the high-dimensional space X on to three dimensions, throwing G and T on to the canonical surface shown in Figure 1 (see Thom. 1972, and Isnard and Zeeman, 1974). Since equivalence preserves qualitative features, and since we only want to discuss qualitative features at this stage, it suffices to look only at the canonical surface.

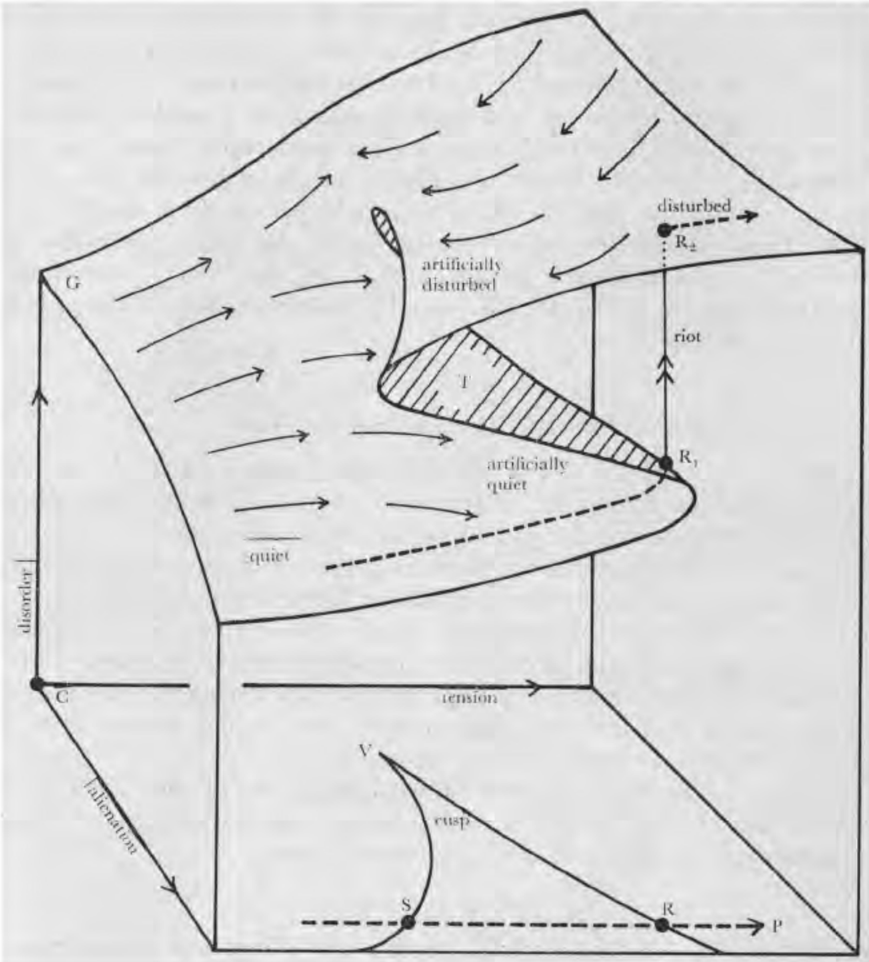

FIGURE 1

Theoretical graph G of disorder as a function of tension and alienation. The dotted path P of continuously rising tension causes a riot at R, where the disorder level jumps catastrophically from the quiet equilibrium at R_1 to the disturbed equilibrium at R_2. The arrows on the surface illustrate the feedback flow.

This means that, if alienation is high and tension moderate, there are two possible levels of disorder if the noise is sufficient to jog the state past the threshold.

The general homeostatic tendency is represented by a flow on G, as shown by Figure 1. We may assume that the dynamic which holds the state on the surface of G is much faster than the flow which moves it slowly about on G in an attempt to restore the homeostatic level. Under conditions of good prisoner-staff relationships, producing low alienation, this flow will

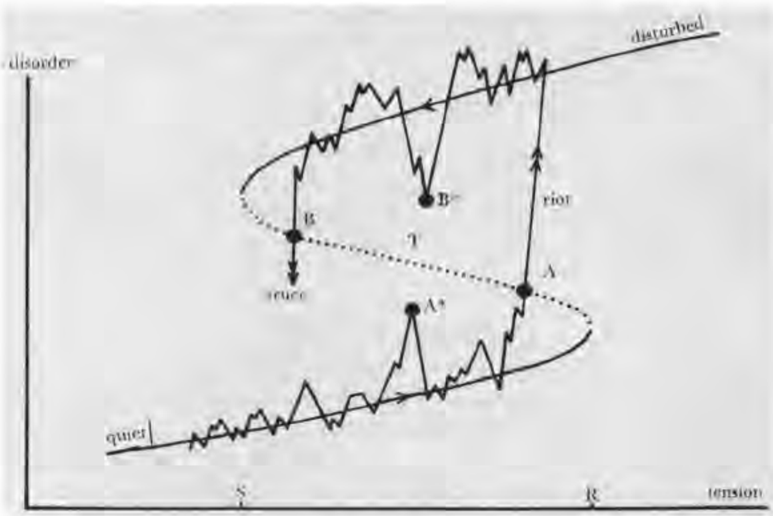

FIGURE 2

Stochastic noise represents incidents that may spark a riot, or moves that may lead to a truce.

therefore contribute to maintaining a long-term stable equilibrium, but with high alienation it will result in catastrophic oscillation between periods of quiet and disorder.

Noise corresponds to random displacements in the three variables and fluctuations on either side of the surface. This is illustrated in Figure 2 which, for simplicity, is a section of Figure 1 at high alienation showing the two variables, tension and disorder. If the state is disturbed by noise, then it is usually restored by the dynamic but noise may trigger a catastrophe by moving the state across one of the branches of the cusp or across the threshold T. For example, in Figure 2, the point A might represent an incident during a period of quiet sparking off a series of incidents leading to a riot. Conversely B might represent a move during a period of disturbance initiating a series of moves, leading to a truce. It will be seen that noise makes the model realistic rather than deterministic—catastrophes occur inside the cusp, just before the cusp lines are reached, rather than on the cusp lines. In the centre of the cusp region catastrophes are *less* likely, as is illustrated by the points A*, B* where noise of the same magnitude as A, B fails to reach the threshold, so the disorder level returns to its previous equilibrium state surface.

Because of the fundamental generality of catastrophe theory, the *qualitative* features of this model should be similar across a wide range of institutions and should be relatively resistant to approximations in measurement. This is fortunate since measurement of the tension, alienation and disorder levels is not easy and one of the usual requirements will probably be the smoothing of raw data to eliminate short-term effects. Since our concern was to develop a method with predictive properties, a *one-sided* exponential filter is indicated. Thus, if Rt denotes the raw variable at time t, the smoothed variable is given by:

$$S_t = (1 - \lambda)(R_t + R_{t-1} + \lambda^2 R_{t-2} + \ldots)$$

where λ is the smoothing coefficient. Since smoothing may conceal critical faster dynamics, it may be more appropriate to use the more powerful Mixed Model smoothing to capture the essentials of both dynamics (Harrison and Stevens, 1971).

Measurement of Variables

Following a series of disturbances at Gartree Prison, data on a number of variables were collected retrospectively for each week of 1972. Collecting data retrospectively is not ideal but as there had been a number of serious disturbances, culminating in a riot in week 48, it was clearly important that an attempt at analysis should be made.

From a principal components analysis of a number of variables which are routinely recorded for administrative reasons by prison staff, a first factor emerged consisting of (i) men reporting sick, (ii) the number of requests to see the Governor, (iii) the number of visits allowed for welfare reasons (although these were incomplete data). Since reporting sick and asking help could both be seen as reflecting, to some degree, tension in an institution, we added the standardised scores together to produce a raw tension variable. Similarly, alienation was estimated by adding together the number of men on punishment and requesting segregation. Both variables were smoothed with $\lambda = 0{\cdot}8$ and the results are presented in Table 1. Although these estimators of tension and alienation are by no means ideal, they appeared to be the best obtainable in the circumstances; measurement deficiencies will, of course, tend to decrease the chance of data fitting our model. It might be thought that segregation might also represent tension and that the number of disciplinary reports should be included in alienation but the relevant data were too uneven to be of use.

Disorder was measured in a rather different way. The list of main incidents during the year is given in Table 2. The seriousness of these incidents was rated on a 10-point scale by seven independent assessors who were staff both of the prison and regional headquarters. As these ratings showed high concordance, the mean value for each week was taken as our measure of disorder.

Analysis

The smoothed values for tension and alienation are plotted week by week in Figure 3 and weeks with a disorder value greater than five have been marked with a circle in Tables 1 and 2 and in Figure 3. Incidents which involved nearly all the prisoners in some new mode of disorder have been marked with a solid circle.

We must first try to locate the cusp. It will be seen that, whilst alienation builds up steadily during the year, tension oscillates to and fro. This may be

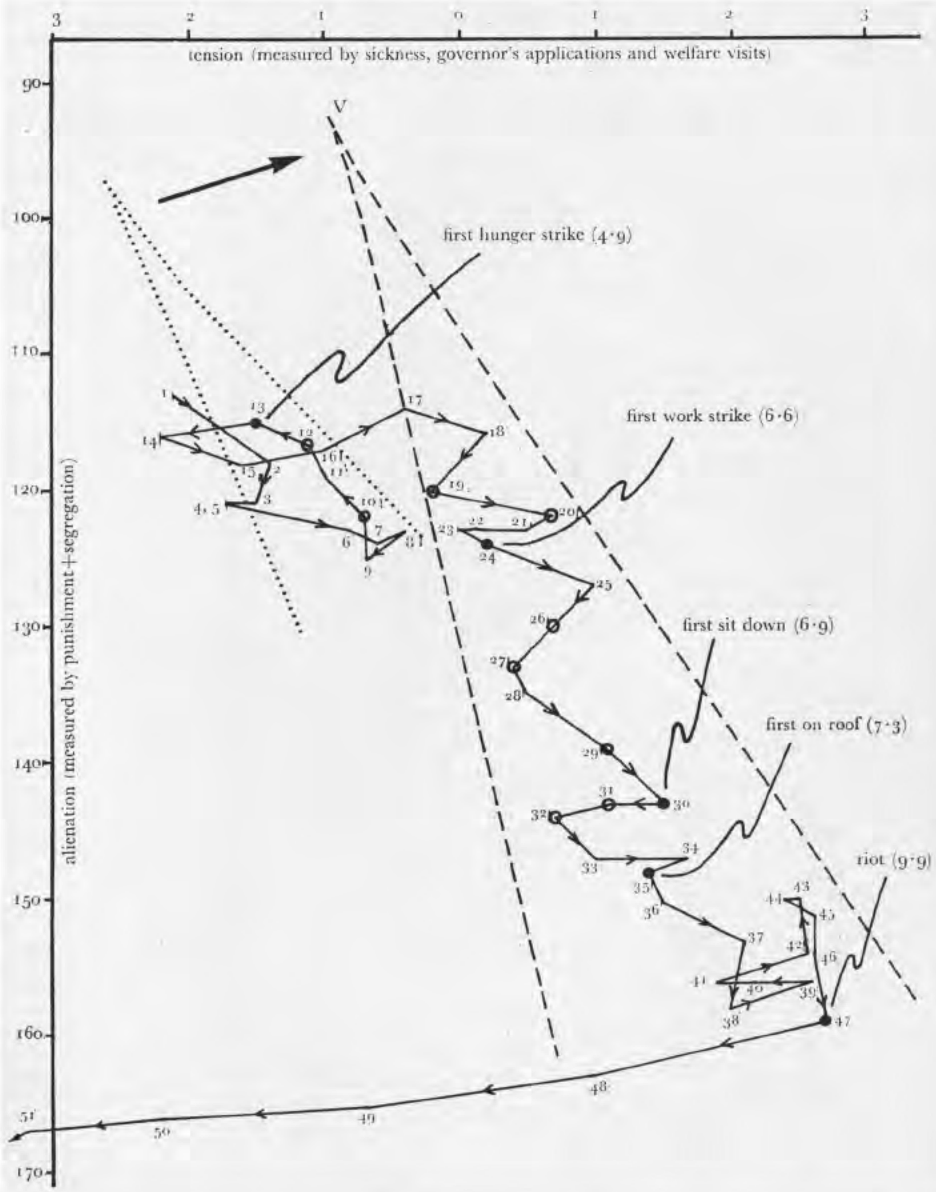

FIGURE 3

Analysis of Gartree data for 1972. Time path of tension and alieniation is plotted weekly throughout the year (numbers indicate weeks). The serious incidents are indicated by circles. The solid circles indicate those incidents involving nearly all the inmates in a new form of mass protest; the numbers in brackets indicate an assessment of seriousness (out of 10). A possible initial position of the cusp is shown dotted and a possible subsequent position is shown dashed; the movement of the cusp may represent a higher tolerance level of tension in the institution after the first mass protest.

TABLE I

Data for tension and alienation

Tension = smoothed standardised sickness + governor's applications + welfare visits

Alienation = smoothed punishment + segregation

Week	Tension Raw	Tension Smooth	Alienation	Punishment Raw	Punishment Smooth	Segregation Raw	Segregation Smooth
1	−3·3	−2·1	113	38	40	73	73
2	1·5	−1·4	118	53	43	82	75
3	−2·0	−1·5	121	56	46	76	75
4	−2·3	−1·7	121	40	45	80	76
5	−1·9	−1·7	121	46	45	74	76
6	2·6	−0·8	123	61	48	71	75
7	0·0	−0·6	124	53	49	77	75
8	0·5	−0·4	123	42	48	74	75
9	−2·1	−0·7	125	46	48	84	77
○ 10	−0·9	−0·7	122	39	46	72	76
11	−2·1	−1·0	119	28	42	81	77
12	−1·7	−1·1	117	27	39	81	78
13	−3·0	−1·5	115	28	37	77	78
14	−5·1	−2·2	116	41	38	77	78
15	0·6	−1·6	118	41	39	82	79
16	1·3	−1·0	117	31	37	85	80
17	2·2	−0·4	114	24	34	78	80
18	2·5	0·2	116	42	36	79	80
○ 19	−2·0	−0·2	120	46	38	90	82
○ 20	4·4	0·7	122	47	40	84	82
21	−0·4	0·5	123	38	40	89	83
22	−1·6	0·1	123	34	39	90	84
23	−0·4	0·0	123	38	39	84	84
● 24	0·8	0·2	124	41	39	87	85
25	4·2	1·0	127	54	42	87	85
○ 26	−0·4	0·7	130	50	44	91	86
○ 27	−0·7	0·4	133	51	45	94	88
28	0·7	0·5	135	51	46	93	89
○ 29	3·5	1·1	139	57	48	97	91
● 30	3·3	1·5	143	57	50	100	93
○ 31	−0·7	1·1	143	46	49	99	94
○ 32	−0·7	0·7	144	63	52	86	92
33	2·0	1·0	147	61	54	96	93
34	4·7	1·7	147	41	51	107	96
● 35	0·4	1·4	148	66	54	86	94
36	2·0	1·5	150	61	55	97	95
37	4·3	2·1	153	72	58	97	95
38	1·8	2·0	158	73	61	103	97
39	4·8	2·6	156	58	60	94	96
40	0·4	2·2	156	69	62	84	94
41	0·5	1·9	156	64	62	94	94
42	5·2	2·6	154	42	58	103	96
43	2·3	2·5	150	35	53	103	97
44	2·1	2·4	150	44	51	108	99
45	3·2	2·6	151	36	48	118	103
46	2·4	2·6	154	37	46	126	108
47	3·3	2·7	159	49	47	127	112
● 48	−5·7	1·0	163	63	50	119	113
49	−6·9	−0·6	165	65	53	106	112
50	−8·6	−2·2	166	56	54	110	112
51	−7·0	−3·2	167	48	53	122	114
52	−7·5	−4·1	172	55	53	138	119

TABLE 2

Disorder: List of the more serious incidents

Week	Date	Assessment	Description of Incident
5	February 4	2·3	Inmate found in possession of counterfeit £5 note.
○ 10	March 10	6·6	Five men involved in assault on officers, then barricaded themselves in recess and smashed it up. All returned to cells in a short time.
○ 12	March 21	6·7	Seven men pushed their way into vestibule (location of special pattern key) and refused to move. Let out through outer door and finally returned to main prison when ordered.
12	March 24	3·9	Assault on an officer.
● 13	March 29	4·9	Hunger strike—total population (c. 370). One meal only. Complaint about food.
19	May 8	5·1	Refusal to work in No. 5 shop (tailors). 40 involved. Going to shop every day but not working. Complaint about rate of pay. Returned to work May 16th.
○ 20	May 17	5·7	Sit-down protest involving 102 men for 12 hours. Complaint about adjudication.
● 24	June 13	6·6	350 men refused lunch, further complaint about food. Work strike (350)—rates of pay. Both ended June 15.
○ 26	June 27	5·7	375 men in work strike—complaint about food. Half day only.
○ 27	July 6	5·3	Fire in tailor's shop store. Extensive damage. Fire brigade needed. Cause unknown.
28	July 14	3·4	Fire in B wing recess. Cause unknown.
○ 29	July 18	5·7	364 men in work strike. Complaint about food. Ended same day.
29	July 22	3·3	Fire in C wing cell. Cause unknown.
● 30	July 26	6·9	378 men in work strike and sit-down in recreation area, Ended July 29.
○ 31	August 4	6·6	Small public demonstration outside prison. After dispersal 208 men sat down in exercise yard and remained until August 5.
○ 32	August 9	6·4	378 men in work strike. Complaint about transfer of a prisoner. Ended August 11.
○ 35	August 29	6·4	364 men in work strike. Complaint about rates of pay and food. Ended August 31.
● 35	September 1	7·3	350 men in work strike. Complaint about adjudication. 211 sat down in exercise yard and 30 climbed on roof. Ended September 4.
46	November 18	3·4	Fire in A wing cell. Fire brigade needed. Cause unknown.
● 47/8	November 26	9·9	13 men in attempted escape. Disturbance developing into violence in C/B dining hall; three fires; disturbance extended to wings. Fire brigade called; police in reserve. Ended November 27.

merely chance variation but the oscillations are precisely what the model leads us to expect through the operation of the homeostatic tendency *within the region of the cusp*. Since we expect catastrophes before the cusp lines are reached, the lines should therefore enclose all the oscillations.

The next task is to locate the cusp point. This is done by plotting disorder against alienation, as in Figure 4. Within the cusp, it can be shown that this section of the surface is a parabola with the vertex at the cusp point. Since the minimum assessment of any reported incident is 2·3, the true homeostatic level of the institution is approximately at level 1: this is the disorder level of the cusp point. If we now draw a parabola through the incidents, paying particular attention to the solid circles with the vertex at

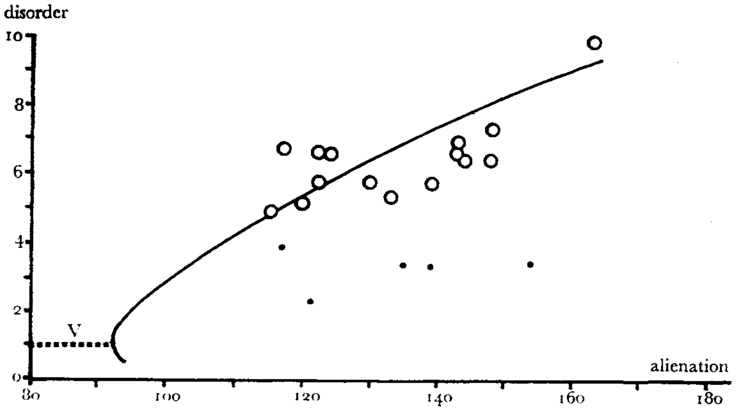

FIGURE 4

Assessment of incidents plotted against alienation. The vertex V of the parabola gives the cusp point.

level 1 (Figure 4), we find that the alienation value of the cusp is between 90 and 100. We can therefore locate the cusp point in Figure 3 and draw the cusp lines at an angle which is roughly bisected by the oscillations.

This position of the cusp fits the data well for weeks 17 to 48 but the initial incidents in weeks 10 to 13 lie outside this cusp. It is possible that the onset of protest in week 13—previously almost unknown in British prisons—affected the institution's level of tolerance of disorder and tension, so stretching the surface and shifting the cusp. Alternatively, our measurement methods may have been too crude to allow proper plotting at the lower levels of tension. Our data do not allow us to choose between these or other possibilities, so we have tentatively drawn a second cusp for the early stages. But we can see emerging many of the features which the model was designed to draw out. It should be remembered that Figure 3 is *three-dimensional*, with disorder in the vertical dimension. During weeks 24–35 disorder is seen to be on the upper, disturbed sheet and during weeks 36–47 on the lower, quieter sheet. We might have expected the final riot a couple of weeks earlier but it is possible that a delay was brought about by other factors to which our measurement was insensitive, particularly the planning surrounding an escape attempt; in any event, two weeks notice of a riot is not without utility. The final " reduction " in tension, following the riot, is very marked. This result is due not only to a change in tension as many institutional processes, *e.g.* welfare visits, were suspended but also to the transfer of some inmates elsewhere.

This is another problem for measurement: activity in the institution during disturbances may alter the inferences one can make from the constituent variables of tension and alienation.

Discussion

We have shown how the application of a mathematical catastrophe model can, even with a retrospective analysis using rather crude data, give a fair account of the circumstances surrounding a series of disturbances and culminating in a riot. Many problems remain to be solved: the two most critical are measurement and some statistical treatment to test the significance or appropriateness of the model. Our reason for presenting the model now is that, in our view, it holds great promise for the analysis of phenomena which have hitherto defied a systematic account.

The importance of catastrophe theory is two-fold: first, it provides the basis for improved *quantitative* monitoring of institutional events and should help in improving the rather rudimentary management information systems currently in use in prisons—when difficulties surrounding measurement, particularly during a series of disturbances, statistical treatment, and the positioning of the cusp (which can be rather arbitrary) are overcome. Normally all one knows is that, as the path wanders east, the nearer catastrophe comes and the more the path moves south, the more pronounced it will be since the upper and lower surfaces diverge along the alienation axis. It follows that the most serious trend is south-easterly since the chance of a dramatic increase in disorder becomes substantial.

Secondly, catastrophe theory can provide *qualitative* insight into the systems susceptible to catastrophic oscillation. The data in this study support the appropriateness of the choice of variables; tension and alienation, in their different ways, do seem to be related to prison disorder.

An example of the way the *qualitative* account given by the model may help in the handling of disorders is given by an analysis of the effect of essentially neutral intervention by institutional management, colloquially referred to as " playing it cool ". With increasing tension and alienation, and therefore unrest, one reaction of management is to increase staffing levels and send staff into the conflict situation with specific instructions to exercise extreme caution. In this way management may be seen as reducing the noise level, so lowering the possibility of a triggering incident. Whilst the resulting confrontation might sufficiently increase communication to reduce tension and alienation directly, it may happen that communication is impossible or the demands of both sides are irreconcilable. The choice management then has is either to wait, adopting a neutral posture, or to restore order by force. The model suggests that, simply by waiting, order is likely to be restored: because of the homeostatic tendency there will be some reduction in tension and alienation, with the time-scale for this reduction being days rather than hours (the flow is slower than the dynamic) but this will eventually cause the state to drop off the upper surface, when order will return. This is illustrated in Figure 2 where B* and B represent the same move designed to achieve a truce, with B* made near the beginning of the unrest and B made after tension has subsided somewhat. B* is unsuccessful because the threshold is too far away but B is successful. It follows that timing is crucial and there is a *minimum period* which *must elapse* before the disturbance can subside, however keen management is to revert to the well-ordered state. As a general observation, prison riots seem to take a few days to settle, whereas hijackers surrender, seem to give themselves up after between 12 and 36 hours. The essential point here is that *because of the characteristics of the model, order is likely to be restored even if prison management appears to do nothing*; it should be noted that, in a conflict situation with tempers running high and everybody rather frightened, it requires skill to remain essentially neutral.

We hope shortly to produce a further paper based on information at Gartree in 1973 and to include in this further discussion of potential operational applications.

Summary

Prison disturbances are often represented as being due to special local circumstances. This paper outlines a general mathematical model for the sudden outbreaks of disturbance in an institution and the model is applied to the events at Gartree Prison in 1972. Whilst the approach is mainly theoretical, some suggestions are made about the handling of disorder, in particular why " playing it cool " is generally likely to be successful.

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