Citation for Samir Siksek (Shephard Prize)

Short citation

Professor Samir Siksek of the University of Warwick is awarded the LMS Shephard Prize for numerous seductively simple and concrete diophantine results whose proofs involve a virtuoso display of the most advanced mathematical ideas.

Long citation

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Many of the theorems he is known for fit effortlessly into the titles of his papers, for instance:

'Every integer greater than 454 is the sum of at most seven positive cubes', *Algebra & Number Theory* 10 (2016), no. 10, 2093-2119;

'Elliptic Curves over Real Quadratic Fields are Modular' (with Freitas and Le Hung), *Inventiones Mathematicae* 201 (2015), 159-206;

'The Asymptotic Fermat's Last Theorem for Five-Sixths of Real Quadratic Fields' (with Freitas), *Compositio Mathematica* 151 (2015), 1395-1415.

Among such a cornucopia of mathematical gems, the Shepherd Prize specifically recognises the works:

- Y. Bugeaud, M. Mignotte and S. Siksek, 'Classical and modular approaches to exponential Diophantine equations I. Fibonacci and Lucas perfect powers', *Annals of Mathematics* 163 (2006), 969–1018.
- Y. Bugeaud, M. Mignotte and S. Siksek, 'Classical and modular approaches to exponential Diophantine equations. II. The Lebesgue-Nagell equation', *Compositio Mathematica* 142 (2006), 31–62.

The first paper proves that the only perfect powers in the Fibonacci sequence are 0, 1, 8, and 14, answering a famous question posed by Mordell 50 years earlier.

The second establishes that the only solutions in integers to the equation $x^2 + 7 = y^n$, with $n \geq 3$ satisfy $|x| = 1, 3, 5, 11$, or 181, resolving an open question formulated in the 1920s, which generalises the famous equation $x^2 + 7 = 2^n$ posed by Ramanujan in 1913 and solved by Nagell in 1948.

The work of Bugeaud, Mignotte and Siksek is notable for combining for the very first time two hitherto disjoint traditions within number theory:

- Diophantine approximation, in the style of Baker, Davenport, Roth and Schmidt. One of the great achievements of this subject, for which Alan Baker was awarded the Fields Medal in 1970, are bounds for certain families of Diophantine equations. Such bounds are however so astronomical as to make complete resolution appear entirely prohibitive.

- Galois representations associated to elliptic curves and modular forms, and in particular the great works of Serre, Ribet, Wiles and Taylor, which led to Wiles' proof of Fermat's Last Theorem.

In the language of the two papers, these are called the 'classical approach' and the 'modular approach' to Diophantine equations.

The papers provide theoretical improvements to Baker's bounds in the critical case of linear forms in three logarithms. They then show how the information provided by the modular approach leads to vast improvements in the bounds: a reduction from doubly exponential to merely singly exponential.

They finally demonstrate how the information from the modular approach can be pieced together to prove that there are indeed no missing solutions.

The papers have been remarkably influential, and many other Diophantine problems have since been successfully attacked by similar combinations of techniques.